The new system of Hyper complex numbers (quart)

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The task was to generalize the usual complex number in case of four-dimensional space, so that each point in four-dimensional space are mutually uniquely correspond to one such generalized number. It was also necessary that the number of newly received had the properties of associativity, commutativity under multiplication and addition. Therefore, the system of quaternions with anticommutativity, is not suitable. On the other hand, the possibility of constructing such a system of numbers implicitly follows from the properties of the so-called Clifford numbers (2), has dimension 2^n.

Let $u = a + bi + cj + dk$ known as quads. In addition, the system quart, of course, has the associativity and commutativity. Consider the multiplication quart.

Let i, j — normal independent imaginary units, $ii = jj = -1$, k is pseudoscalar, $kk = 1, ik = ki = j, jk = kj = i, ij = ji = -k, ijk = -1$. Consider table 1 multiplication quart:

Table 1

	1	**i**	**j**	**k**
1	1	i	j	k
i	i	-1	-k	j
j	j	-k	-1	i
k	k	j	i	1

Note that the multiplication chain has the properties of associativity and commutativity, which allows their use in algebra and possibly in the analysis. Non-negative real number r =$r = \sqrt{a^2 + b^2 + c^2 + d^2}$ let's call the norm quarts.

If $u = a + bi + cj + dk$ is the algebraic form of quarts, then u=r $(\frac{a}{r} + \frac{b}{r}i + \frac{c}{r}j + \frac{d}{r}k)$ $= r(\cos\varphi_1 + \cos\varphi_2 i + \cos\varphi_3 j + \cos\varphi_4 k)$ can be understood as the trigonometric form of quarts. Will write down the generalized Euler formula:

$$e^u = e^a \times (\cos\alpha_1 + i\sin\alpha_1) \times (\cos\alpha_2 + j\sin\alpha_2) \times (ch\alpha_3 + ksh\alpha_3) =$$

$$e^a \times [(\cos\alpha_1\cos\alpha_2 ch\alpha_3 - \sin\alpha_1\sin\alpha_2 sh\alpha_3) + i(\sin\alpha_1\cos\alpha_2 ch\alpha_3 + \cos\alpha_1\sin\alpha_2 sh\alpha_3) +$$

$$+ j(\cos\alpha_1\sin\alpha_2 ch\alpha_3 + \sin\alpha_1\cos\alpha_2 sh\alpha_3) + k(\cos\alpha_1\cos\alpha_2 sh\alpha_3 - \sin\alpha_1\sin\alpha_2 ch\alpha_3)]$$

Quadratic equations in dollars will have 4 solutions of equations of degree n — 2n solutions. Examples:

$x^2 = 1$; $x_{12} = \pm 1$; $x_{34} = \pm k$;

$x^2 = -1$; $x_{12} = \pm i$; $x_{34} = \pm j$

$x^2 = i$; $x_{12} = \pm \frac{1}{\sqrt{2}}(1+i)$; $x_{34} = \pm \frac{1}{\sqrt{2}}(j+k)$;

Note that the decomposition into a work program for fast polynomial is not the only one.

You can consider the properties of elementary functions of a variable in dollars.

1. The power function.

Let $u = x_1 + x_2^i + x_3^j + x_4^k$, $w = y_1 + yx_2^i + y_3^j + y_4^k$— variables in dollars, $w = U^n$ (n-positive integer). It appears that the power function is defined for all quart, each number it puts u in a match U^n . Many quarts, norm equal to 1 is represented by a sphere of radius one in four-dimensional coordinate system.

2. The exponential function.

This function has the form:

$$w= e^u = e^a \times (\cos\alpha_1 + i\sin\alpha_1)\times(\cos\alpha_2 + j\sin\alpha_2)\times(ch\alpha_3 + ksh\alpha_3) =$$

$$e^a \times [(\cos\alpha_1\cos\alpha_2 ch\alpha_3 - \sin\alpha_1\sin\alpha_2 sh\alpha_3) + i(\sin\alpha_1\cos\alpha_2 ch\alpha_3 + \cos\alpha_1\sin\alpha_2 sh\alpha_3) +$$

$$+ j(\cos\alpha_1\sin\alpha_2 ch\alpha_3 + \sin\alpha_1\cos\alpha_2 sh\alpha_3) + k(\cos\alpha_1\cos\alpha_2 sh\alpha_3 - \sin\alpha_1\sin\alpha_2 ch\alpha_3)]$$

$$= e^a(\beta_1 + \beta_2 i + \beta_3 j + \beta_4 k.$$

Note that $\beta_1^2 + \beta_2^2 + \beta_3^2 + \beta_4^2 = ch^2\alpha_3 + sh^2\alpha_3$. Let $\rho = \sqrt{ch^2\alpha_3 + sh^2\alpha_3}$.

$$r = e^{a} \cdot \rho$$

$$\cos\varphi_1 = \frac{\beta_1}{\rho}$$

Then $\cos\varphi_2 = \frac{\beta_2}{\rho}$ the formulas of transition from exponential form quarts to

$$\cos\varphi_3 = \frac{\beta_3}{\rho}$$

$$\cos\varphi_4 = \frac{\beta_4}{\rho}$$

trigonometric.

For quart is Euler's formula, the function $w = e^{n}$ is periodic. Similarly, we can introduce trigonometric, hyperbolic, inverse trigonometric and hyperbolic functions. Further, we can introduce the notions of limit and derivative of a function in dollars, if there are any difficulties. In conclusion, we hope that other such elementary properties quarts will be investigated and described.

Literature

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