SYSTEMS WITH MULTIPLE IMPACT PAIRS AND NON TADITIONAL DISTRIBUTED IMPACT ELEMENTS

V.L.Krupenin

Mechanical Engineering Research Institute
4, M. Chariton'evsky per, Moscow, Russia, 101990.
E-mail: krupenin@online.ru

In this review in general the dynamic effects are considered and described, caused by different standing waves in discrete and distributed systems with impacts and other strongly positional non-linear factors, introduction of which in the models of mechanical objects is dictated by the necessity to consider processes, accompanied with impacts of mechanical subsystems of different nature. This is traditional and non-traditional vibro-impact systems such as, for example, mechanical filters of high and low frequencies, which elements suffer impacts; various extended constructions, vibrating near point-mass and T-obstacles, lattices and continuous movement restrictions; vibro-conductive and vibro-arresting constructions, equipped with facilities with solitary and multiple breaks, etc. We examined also dynamics of a complex system consisting from any number elastic - viscous subsystems interacting through impacts are studied through techniques of a frequency-time analysis. This research consists of several parts.

1. Only few words about frequency-temporal technique
2. Systems with serial impact pairs
3. Systems with parallel impact pairs
4. Systems with distributed impact elements.
5. Some results of the experiments (waves picture).

Author tried to present the material within the unified pattern: basic model, describing groups of objects; examples of motion equations; methods of its analysis; dynamic effects; results of experiments. The received solutions are analytical. Different calculation methods are presented, based on frequency-temporal analysis and other methods of the modern non-linear mechanics.

Some experimental plants were designed and are created (together with the Dr A. Veprik, Prof V. Astashev, Dr A. Tresvjatsky and Mr. A. Sternin) for realization of full-scale experiments on some basic models. As a rule literary references in the text are not resulted in generally. (See general observations at the end of the paper).

1. Only few words about frequency-temporal technique for analyse vibro-impact (Babitsky and Kolovsky, 1974, Krupenin 1979 – 1985)

These methods for systems with a single impact pair were developed on monograph , where the two-parametric representation of T-periodic vibro-impact process with a single impact during the motion period was detail examined.

Let unknown vibro-impact process is u(t). And let in absence of impacts this process is $u_0(t)$. Under the assumption of existence of T-periodic vibro-impact motion mode the operator equation of motion can be written as follows

$$u(t) = u_0(t) - \int_0^T \chi(t-s)\, \Phi(u,u_t)\, dyds, \qquad (1)$$

where Φ - force of momentary impact understood in sense of distributions; the periodic Green function (PGF)

$$\chi(t)=T^{-1}\sum_{k=-\infty}^{\infty} L(ik\omega)\exp(ik\omega t)$$

is a steady-state response of a linear system with operator L(p) to a force action represented by T-periodic sequence of Dirak δ-functions:

$$\delta^T(t)=\sum_{q=-\infty}^{\infty}\delta(t-qT).$$

By assuming that point wise bodies with finite masses directly participate in collisions and, consequently, an impact can be described by the Newton hypothesis, and supposing the process to be T-periodic, we obtain: $\Phi[u(t),u_t(t)]=J\delta^T(t-\varphi)$,where J designates impulse and a φ- phase of the impact.

On substitution of (12) into equation (11) we obtain the desired two-parametric representation

$$u_j(t) = u_0(t) - J\chi(t-\varphi), \quad (2)$$

The unknown J and φ can be sought from the impact conditions

$$u(\varphi)=\Delta,\ \dot{u}(\varphi+0)=-R\dot{u}(\varphi-0),\ J= m(R+1)\dot{u}(\varphi-0). \quad (3)$$

where R - coefficient of velocity restoration at impact, which here is supposed to be straight and central; Δ - set-up clearance (interference) value; $m = m_1m_2/(m_1+m_2)$ - reduced mass of colliding bodies.

The solution (2) should be tested for feasibility of geometric condition $u\leq\Delta$ and stability. It's worthy noting that the representation (2) gives an opportunity to build up exact motion manners for any element of a rather generalized system without resorting to joining of solutions procedure.

We also emphasize that this representation allows for multiple useful modifications. And the systems with many impact pairs or distributed impact elements were investigated. However, the more complex (but similar!) representations (multiparametrical, multifunctional, etc.) were used.

2. Systems with Serial Impact Pairs

2.1. Basic models.

Systems with serial impact pairs include vibro-impact systems with large numbers of degrees of freedom in which all elements except for outer ones are involved in two or more impact pairs.

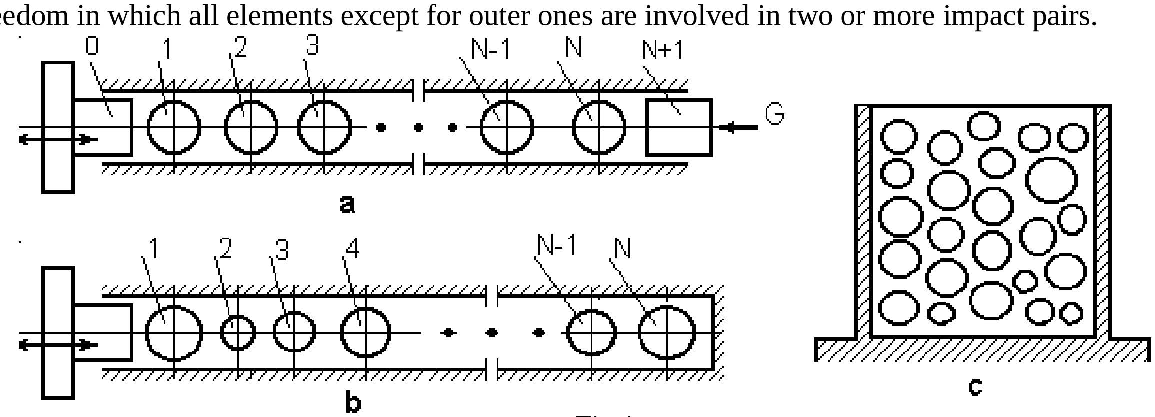

Fig.1

Some of the typical models of such systems are shown in fig.1.

The model in fig.1,a contains N balls of the mass m, placed between the zeroth and (N+1)th elements. Element 0 is being cinematically put in motion with a given amplitude, and (N+1)th element of a mass M is either fixed rigidly or loaded with a constant force Q. The one-dimensional model in fig.1,b differs in that the bodies in impact pairs have different sizes and masses. Some of the bodies can be linked with each other and the base by elastic elements. The models presented contain N+1 impact pairs. Systems with serial impact pairs include also spatial structures similar to that shown in fig. 1,в, where various bodies are contained in a closed vibrating volume.

2.2. Motion equations for systems with serial impact pairs.

A number of systems with serial impact pairs were examined in the works [2,6], where periodic motions in one-dimensional basic systems of the type as shown in fig.5,a,b, are analyzed. Motion of the j-th element during the time interval between collisions is given by the equation

$$m_j\ddot{x}_j + b_j\dot{x}_j = 0. \qquad (4)$$

Equations (20) should be added with conditions in k-th impact pair at the moment of a collision $t=t_k$ in the form (3), in which u value should be replaced with $u_k = x_k - x_{k-1}$, supposed $\varphi = t_k$ and assumed $u_k(t_k)=0$ instead of the first condition.

Evidently, equations for impact elements of the type (4) can be significantly generalized. However in such case motion of a chain with serial impact pairs may become extremely complex.

2.3. Analysis methods.

Systems with serial impact pairs were examined in [4,11] using the fitting technique. In so doing one-dimensional chains of point bodies are considered and it is assumed that energy dissipation occurs only as result of impacts. The analysis is simplified by the obvious condition that in periodic modes impact forces pulse value in all impact pairs J=const. For chains of the type like in fig.1,a with force closure the pulse value J=QT (T - process period), and dynamical length of a chain is set to provide this preliminary determined impulse. For chains with a given length impulse value can be found as a solution of a periodic problem.

2.4. The essential dynamic effects.

It is easy to find out that in a one-dimensional chain of balls with equal masses m and absolutely elastic impacts the pulse J imparted to the first ball propagates along the chain undistorted as a peculiar kind of diffusive wave of the impact force $\Phi(t,x)=J\delta(vt-x_j)$, where $v=J/m$; x_j - coordinates of initial locations of the balls at rest; j - number of a ball in the chain. Evidently, this wave can reflect from a stationary barrier changing the propagation direction. The relationship between this condition and existence of solutions like solutions in different vibro-impact systems with multiple degrees of freedom was discussed, particularly in [14].

3. Systems with Parallel Impact Pairs

3.1. Basic models.

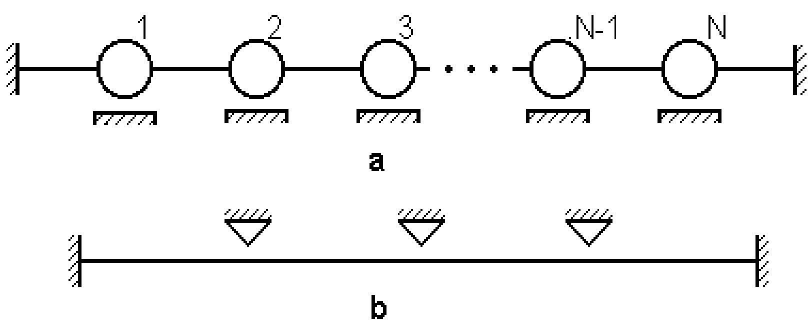

Fig.2

Systems with parallel impact pairs include complex vibro-impact systems, in which some of the elements of one (basic) subsystem constitute impact pairs with elements of other subsystems, and each element of the basic subsystem can be incorporated in only one impact pair. Examples of such systems are shown in fig.2.

Fig.2,a shows a system with basic subsystem presented by a string with N balls fixed on it. In this case the balls collide with a rigid one-side restrictions, which constitute the second subsystem. Naturally, the restriction may also be double-sided. Besides that, the balls may constitute impact pairs with elements of more complicated form. Fig.2,b displays a transversely oscillating string or beam interacting with point-wise restrictions. Here the subsystem with distributed parameters acts as a basic.

3.2. Motion equations for systems with parallel impact pairs.

Using the dynamic characteristic of an impact pair obtained earlier, we write motion equations for a system with N parallel impact pairs (fig.2,a)

$$m_1\ddot{u}_1+b_1\dot{u}_1+(c_1+c_2)u_1-c_2u_2+\Phi_1(u_1,\dot{u}_1)= P_1(t),$$

$$m_j\ddot{u}_j+b_j\dot{u}_j+c_j(u_j-u_{j-1})+c_{j+1}(u_j-u_{j+1})+\Phi_j(u_j,\dot{u}_j)=P_j(t),\ j=2,N-1 \qquad (5)$$

$$m_N\ddot{u}_N+b_N\dot{u}_N+(c_N+c_{N+1})u_N-c_Nu_{N-1}+\Phi(u_N,\dot{u}_N)=P_N(t),$$

where u_j - coordinate of the j-th body; m_j - it's mass; c_j – stiffness of the j-th string portion; b_j - coefficient of resistance to the motion of the j-th body; $P_j(t)$ - inducing force affecting the j-th body (j=1,...,N); $\Phi_j(u_j,u_j)$ - dynamic characteristic of the j-th impact pair.

Motion equation for a system with distributed parameters, in which inducing forces $P_k(t)$ act, for instance, in points of location of impact pairs x_k has the following form:

$$\rho u_{tt}-C^\wedge[u]+B^\wedge[u]=\sum_{j=1}^{N}[-m_ku_{tt}(x,t)+ P_k(t)- \Phi[u_k(x,t),u_t(x,t)]\delta(x-x_k), \qquad (6)$$

where C^ and B^ - linear "elastic" and "dissipative" operator of the system; r - linear density of the material, m_k - masses of the bodies located in points x_k.

Independent of the system structure the equations (16), (17) etc. may be written uniformely in the operator form. For the required movements field u(x,t):

$$u(x,t)=\sum_{k=1}^{N} L(x,x_k;p)[P_k(t) - \Phi(u_k,pu_k)], \qquad (7)$$

where the dynamic compliance operators L(x,y;p) are determined by the structures of the initial interacting subsystems.

For description of the system (16) with concentrated parameters in the operator equation (18) we should set $x=x_j$.

3.3. Analysis methods.

The study of systems with parallel impact pairs is carried out numerically or by means of frequency-temporal analysis methods.

In case of a periodic outside excitation in order to find the T-periodic modes instead of (18) we can obtain:

$$u(x,t) = u_0(x,t) + \sum_{k=1}^{N}\int_0^T\chi(x,x_k;t-s)\Phi(u_k,u_k)ds, \qquad (8)$$

where $u_0(x,t)$ - steady-state movement field under inducing forces in the absence of impact interactions; $\chi(x,x_k;t-s)$ - periodic Green function, compliant to the operator $L(x,x_k;p)$ (built up similarly to part); see also).

If assumed that like above the impact is momentary and acts one time during the motion period, so that the function $\Phi(u,u_t)$ can be presented as a combination of singular distributions, the equations (19) can be reduced to the following representation of the vibro-impact process:

$$u(x,t) = u_0(x,t) + \sum_{k=1}^{N} J_k\, \chi(x,x_k;t-t_k) \qquad (9)$$

where J_k - impact forces impulse in the k-th impact pair; t_k - moment of the impact in this pair. Representation (9) by analogy with the main representation (13) is called "2N-parametric". The unknown motion parameters can be sought from the impact conditions

$$u(x,t_k)=\Delta_k, \quad J_k= m_k(R_k+1)u_{kt}(x,t_k-0),$$

where Δ_k, R_k - clearance value and velocity recovery coefficient in the k-th impact pair.

The solutions obtained should be analysed regarding the stability and feasibility of geometric conditions of the type $u_k(x,t_k)\leq\Delta_k$.

The final solution of the problem in a visible analytical general form can be obtained for a limited number of models of such kind, however, for particular parameters it is almost always possible to find a corresponding numerical-analytical solution. Besides that, basing on the representation (20) it is possible to build up some approximate solutions.

3.4. The essential dynamic effects.

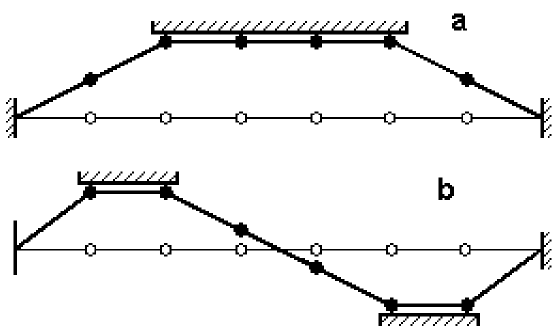

Fig.3

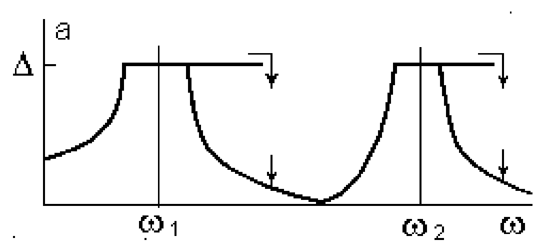

Fig.4

Further we will discuss some effects revealed as a result of analysis the model (5) with a periodic structure: for each j all m_j, b_j and c_j values are the same and equal to m, b, and c correspondingly. The outside excitation was chosen sinusoidal.

The main result (see also clause 4.4) is the discovery of existence of periodic modes with synchronous impacts in remote impact pairs. Such modes were calls “claps” or “puffs”. At excitation of these modes the string with the balls fixed on it generates one-, two- or multitrapezoid forms quite similar to the corresponding inherent oscillations forms of a linear system in terms of alternation of nodes and crests of waves, and feasible in frequency areas situated at the right from the inherent frequencies of the corresponding linear system. Fig.3 shows an example of claps for two lowest forms in the system with six symmetrical impact pairs.

The trapezoid form appears even with two impact elements. This is not surprising, since in this case exactly this form is the first inherent form of linear system oscillations. The same form with two elements in the crest of the wave is one of the higher inherent forms for systems containing 6n-1 elements, n=1,2,... and therefore for them realization of such form in vibro-impact processes is quite natural. The surprising thing is that trapezoid form become dominating as the number of elements in oscillations of the crest of the wave increases.

At realization of one- and multitrapezoid standing claps-waves the dynamic effects arise which are typical of systems with a single impact pair: phenomena of delaying by frequency and amplitude, and also hard initiation (see clauses 4.4 and 5). Thus the behavior of claps-waves is in many respects similar to the behavior of a system with a single impact pair and, particularly, of a simple "impact oscillator".

Fig.4 displays the amplitude-and-frequency characteristic of a system with parallel symmetrically situated restrictions in a neighborhood of two lower inherent forms of the linear system. At amplitudes which are less than the clearance ($a<\Delta$) the branches of resonance curves of the linear system are being realized. After reaching of the restrictions a vibro-impact process of the puff type takes place, number of bodies involved in the puff depending on the inducing forces level. The figure illustrates the above-mentioned analogy between dynamics of puffs and vibro-impact systems with a single impact pair.

Besides the above mentioned, these "similar" effects include, in particular, appearance of ambiguity areas of amplitude-frequency characteristics and breaking of oscillations away from one stable branch of resonance curve to another (arrows in fig.4).

It should be noted that distributed systems with multiple parallel impact pairs like shown in fig.4 have similar characteristics.

The numerical analysis made it possible to find in these systems modes of significantly more complicated nature, which can be considered as chaotic.

Note, that the models of systems with distributed impact elements discussed below, in many cases can be considered as long-wave approximations to discrete models of the type (5) (see below).

4. Systems with distributed impact elements

4.1. About this systems.

4.1.1. In this chapter we will briefly examine the problems concerned with the models of systems with distributed impact elements, which can appear while studying discrete systems of the type (16) as the corresponding long-wave approximations, or while using axiomatic approaches.

It is reasonable to consider distributed impact elements, particularly, when studying vibro-conductive (vibro-inhibiting) continuous media with complicated structures, and also when studying various extended objects (strings, threads, flexible bars, cables, ropes, beams and other systems), moving near some solid or latticed obstacles, partitions, walls, grids, etc.

4.1.2. References considered models of distributed linear media of complicated structure. The most specific feature of mentioned structures is the presence of two main "medium parts", namely: "carrier" and so called "attached" parts. Dynamic description of such systems, in general, consists of two groups of motion equations - reflecting "carrier" and "attached" subjects constrained behavior respectively. Likewise all the models in the multipolar mechanics, the concept of a point is a subject of significant revision: its state can be defined by unspecified number of kinematic parameters.

Such models utilization arise to be productive for solution of some practical problems, such as dynamical analysis of vibro-states of complicated mechanical structures, consisting of, said, distributed single dimensioned "carrier" and of gross number flexural "attached" solid devices. Monograph, particularly, considered models of media with non-linear damping.

At the same time, paper considered the problem of random oscillations of a distributed carrier rod with gross number of separated impact pairs being attached flexural along it.

It seems that mentioned considerations could be useful by dynamic models creation for the systems consisting of "carrier" and "attached" parts with multiply breaks in it. Due to the system nature the possibility of different type collisions arises in the "attached" subsystems.

Then, unlike the approach of where it was assumed that the impacting elements concentration is low, we consider completely distributed model. Thus, assuming the impact pairs to be "spreader" within a certain space, we can use the concept of distributed impact elements.

4.1.3. Now, let's overview briefly some papers concerning the distributed impact elements. We can obtain the model of distributed impact element by at least two ways.

Firstly, in some cases it appears impossible to disregard wave process arising in the impact pairs itself. Impacting bodies can't be considered as the solid bodies since the lengths of the waves generated by collisions are comparable with the impacting surfaces dimensions.

Secondly, considering the dynamic system with amount of convenient impact pairs to be large enough we can perform long-wave approximation and transit to the distributed model with distributed impact element.

In this way, starting with a abstract model of "regularly" concentrated beds on the strained thread Φ colliding with the solid stationary obstacle we can obtain basic physical model of the string, colliding with the solid stationary obstacle (see fig.5.a,b).The number of technical applications for such a model is huge. The mentioned above basic model, initially considered in the paper of L.Amerio and G.Prouse , is now examined deeply enough theoretically in the papers and experimentally (further references may be found in above papers).

4.1.4. The idea to use the concept of distributed impact elements seems to be also productive for dynamic analysis of great number of lightly damped vibro-isolated devices attached over the flexible carrier with the motion limiters and interacting dynamically via carrier (see fig.6,7).

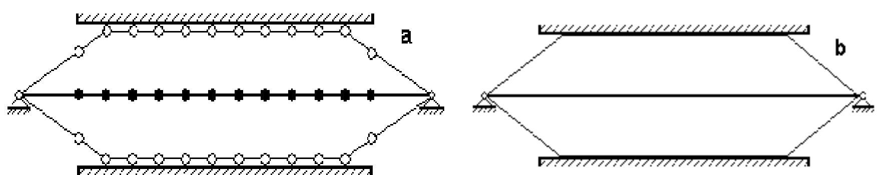

Fig.5

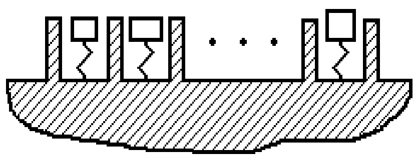

Fig.6

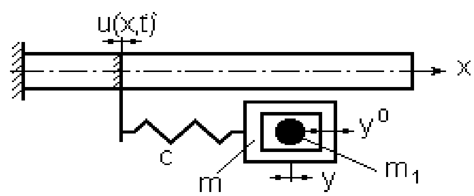

Fig.7

This model is an exact model of strongly non-linear vibro-conductive media with complicated structure. Evidently, for the first time this problem was considered in. In this paper the problem of vibration propagation through a strongly nonlinear vibro-conductor (bar with the distributed impact element, modeling the two degree of freedom attached equipment) was solved. Main definitions and solution method based on so-called non-linear forms of oscillations (periodical motion conditions having certain symmetric properties) and generalized methods of frequency-time analysis of vibro-impact processes were suggested.

4.2. Motion equations for systems with distributed impact pairs.

4.2.1. Let's postulate the existence of some elastic carrier medium, which motion being described by function $u(x,t) \in R^3$ ($x \in R^3$, $t \in R$), subjected to the Lame equation

$$\rho u_{tt} = (\lambda + \mu) \text{ grad div } \Delta u + \Delta u + F \qquad (10)$$

where ρ - density of the carrier medium and Lame factors λ, μ characterize the carrier elasticity properties.

Let the intensity of the volumetric forces has the following structure: $F= F_0+ F_1$ ($F\in R^3$), where F_1 - dictated vector, and F_0 (x, t) – vector-function describing influence of the vibro-impact processes generated in the attached equipment.

For example - the longitudinal waves propagation in the rod with flexural attached in its every point impact pair (see fig.6). Applying above-mentioned approach we can obtain the following system of equations:

$$\rho u_{tt} - EAu_{xx} +c(u - y)=0, \quad my_{tt} + c(y - u) - \Phi(y^0) = 0, \quad m_1(y_{tt} + y^0{}_{tt}) + \Phi(y^0) = 0, \qquad (11)$$

where u,y,y^0 – are the fields of displacements of system structural elements with current coordinate x; u=u(x,t) - carrier displacements; y=y(x,t) and $y^0=y^0(x,t)$ - displacements of attached elements; ρ, A, E– linear density, cross sectional area and Young modules of rod material respectively; $\Phi(y^0)$ - impact force density; m, m_1 - linear densities of attached parts. Boundary conditions for (11) looks like: u(0,t)=0, $u(l,t)=\mu\cos\omega t$, where l - the length of the rod. The analog of Newton's conditions impact are as follows: $y^0[x,\varphi(x)]=\Delta$, $J(x)= M(1+R)y_t{}^0[x,\varphi(x)- 0].0$, where Φ(x) - the impact force phase distribution, J(x) - the impact force impulse distribution, $M = mm_1/(m+m_1)$ - reduced mass distribution, R - restitution ratio.
In the mostly general case: Δ=Δ(x), R=R(x), A=A(x), M=M(x), but the system description is still the same. In the similar manner the models for the beams, membranes and plates may be considered.

The boundary conditions for these types of models the carrier construction (namely, physical and geometrical characteristics) are the same as in the classical case. The frequency properties of the amortized equipment generating vibro-impact processes can be obtained from the model of the added medium part, containing the impact element. The connection mechanism of the carrier and added parts determines the structure of global vibration field. This approach, adapted for analysis of the collective effect of impact pairs, possibly sacrifices information on "individual" features of certain concrete system elements, as well as effects, which can only be clarified if considering the model's discreteness .

4.2.2. Let's consider vibrating string or supported flexible beam, colliding with obstacles of various kinds. For example, using the Timoshenko beam model and denoting by u(x,t) and y(x,t) - $u_x(x,t)$ the beam instantaneous linear and angular deflection shape, Φ(u) impact force distribution, the motion equations will be written as following: $\rho U_{2t} - k'FGu_{2x} + k'FGy_{2x}+ \Phi(u) = P(x,t)$; $E\Gamma y_{2x}- k'FGu_x - \Gamma\rho F^{-1}y_{2t} = 0$ with the boundary conditions u(0,t) = u(l,t) = 0 , etc. $0<x< l$. $-\infty<t<\infty$. Singular function Φ(u) is defined by type of the obstacle and properties of model. The standard set of parameters and modules enters into equation Timoshenko.

4.3. On the analysis methods

In spite of the complexity of the motion equations, in some cases it appears possible to perform not only the corresponding numerical analysis of mentioned models, but to obtain an approximate analytical representation of the required motion distributions as well. These representations can be obtained by means of the modified frequency-time analysis of vibro-impact processes and other methods of the modern non-linear mechanics. In many cases the equations of motion can be reduced to the following "two-functions" representation of the vibro-impact process [u(x,t)]:

u(x,t)=u0(x,t)-òJ(z)c[x,z;t-f(z)]dz,

X

where $u_0(x,t)$ - steady-state movement field under inducing forces in the absence of impact interactions; X – some integration area ; χ(x,z;t) - PGF compliant to some operator L(x,z;p) (built up similarly the motion equation). Unknown two functions J(x) (the impact force impulse distribution) and f(z) (the impact force phase distribution) can be sought from the impact conditions.

4.5. Essential dynamic effects.

Calculations of a number of concrete systems enable to find out and systematize various dynamical effects arising there. Let's note the most specific and impressive ones:

4.5.1. Vibro-conductors with impact elements:

- advent of spatial areas of the most intensive collisions;
- advent of spatial areas of "transparency" and "locking" of vibro-conductive media for the main oscillation tone. The structure of these areas can be very complicated and determined by some specific resonance relationship depending upon the physical and geometric properties of the carrier part of the vibro-conductor, upon the structure and frequency properties of the attached equipment, containing certain types of impact pairs, and, of course, upon the dissipate factors;
- generation of the higher harmonic components of propagating vibration, including the case of a main tone delay;
- advent of combined (particularly, sub-harmonic) motion conditions;
- advent of chaotic motions.

4.5.2. One-dimensional extended objects (strings, beams, etc.) vibrating near straight obstacles:

- arising of trapezoid standing waves ("claps"), characterized by synchronous coming of distant points of distributed systems to the restrictions;
- emergence of higher forms of puffs (multitrapezoid standing waves);
- arising, as the puffs occur, of the effects, characteristic to "impact oscillators": "delaying", multivaluedness of "amplitude"-and-frequency characteristic, feasibility of "hard excitation" and others;
- arising of near-periodic standing waves;
- retention of trapezoid profiles of standing waves at different kinds of outside and self-running excitation;
- arising of standing waves characterized by specific profiles of complex nature (quasi-adhesion, emergence of "inside out" configurations etc.)

5.Some results of the experiments (waves picture)

5.1. The experimental study of systems with distributed impact elements

The experiments with a distributed impact element were carried out at the Vepik stand schematized in fig.8. Here a rubber tourniquet, one end of which is connected to the force sensor FS fixed on the carriage K1, and the other is linked with the rod of electro-dynamic vibrator B, was used as a distributed impact element РЭ. The tourniquet tension can be adjusted by moving the carriage C1 using the screw S1. It's oscillations are restricted by the extended plate П, fixed on the carriage C2, and moving it with the micrometric screw S2 enables to change the clearance between the tourniquet and the restriction. The vibrator V is energized by the control generator of sinusoid oscillations CG.

The signal from the force sensor FS, proportional to the angle of cord rotation was registered by the electronic rays oscillograph O. The tourniquet's standing waves were observed in stroboscope light, generated by movement analyzer MA, which lamp's L bursts are synchronized with the control generator. The phase-rotator built in the analyzer enables to stop and photograph (with camera C) the tourniquet's form at any movement phase, and setting of a small detuning between the frequencies of bursts and excitation enables to observe a slowed-down display of evolution of standing waves.

The stand allows setting the second restriction (not shown in fig.13), which enables to examine systems with both one-side and two-sides restrictions of the string oscillations.

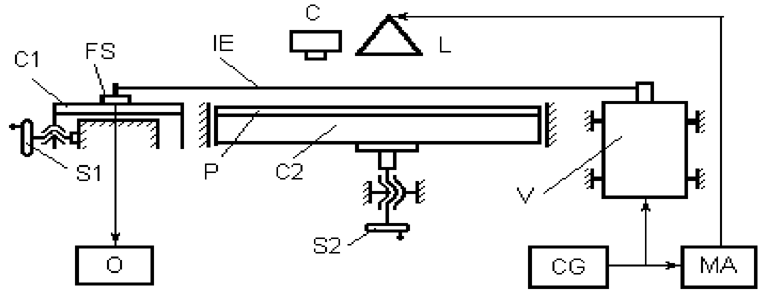

Fig.8

We cite some results of the examination of periodic standing waves, observed at this unit.

At excitation frequencies $\omega<\omega_1<\Omega_1$, where Ω_1 - the first inherent frequency of the linear system, w_1 - the frequency at which pre-resonance branch of amplitude-and-frequency characteristic passes the restriction level, sinusoid standing waves arise without contacts with the restriction and with amplitudes within the clearance.

In the range $\omega_1<\omega<\Omega_1$ the waves were found called "on-running without rebound". These waves are different in that the string points reaching the restriction stop immediately and rest at it for some time. As this happens, the wave running on the restriction "spreads" along it until the string takes a certain "final" configuration. Then the string points leave the obstacle and the process recurs.

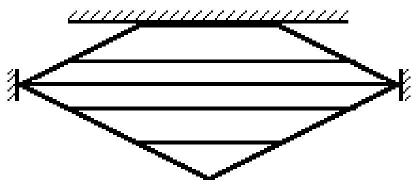

Fig9,a

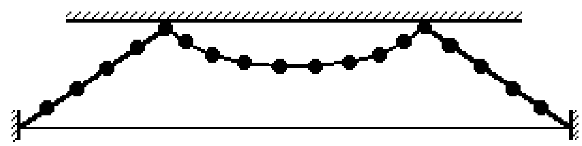

Fig.9,b

At passing of the linear resonance frequency $\omega=\Omega_1$ on-running waves disappear and resonance modes of the "claps" type arise. At puffs points of a certain tourniquet part concurrently reach the restriction and rebound immediately. The characteristic standing wave evolution in a system with one-side restriction is shown in fig.9, where it is evident that intermediate trapezoid wave configurations at the end position degenerate into an isosceles triangle, as predicted by the theory. Claps are excited in the frequency range $\Omega_1<\omega<\omega_*$, where $\omega_*>\omega_2$, ω_* - the ultimate puffs excitation frequency, ω_2 - frequency, at which the over-resonance branch of the system's resonance curve crosses the restriction level. In the area $\omega_2<\omega<\omega_*$ in par.

At the same time with claps there exist sinusoid standing waves within the clearance without contacting the restriction. Claps-type modes are of a clearly defined non-linear resonance nature; the essential non-linear effects typical for an ordinary impact oscillator are characteristic for these modes: delaying by frequency and amplitude, quenching of oscillations, feasibility of hard starting of vibro-impact modes.

Similar effects were also observed in the neighborhood of higher forms of linear system oscillations when either one-side or two-sides restrictions were installed (see 4.5.2). And also similar effects were observed in case of lattices, point-mass obstacles (on fig.10 the string in various phases of movement in systems with one and two obstacles is shown) and at interaction with T-obstacles (the typical structure of a standing wave is shown on fig.11).

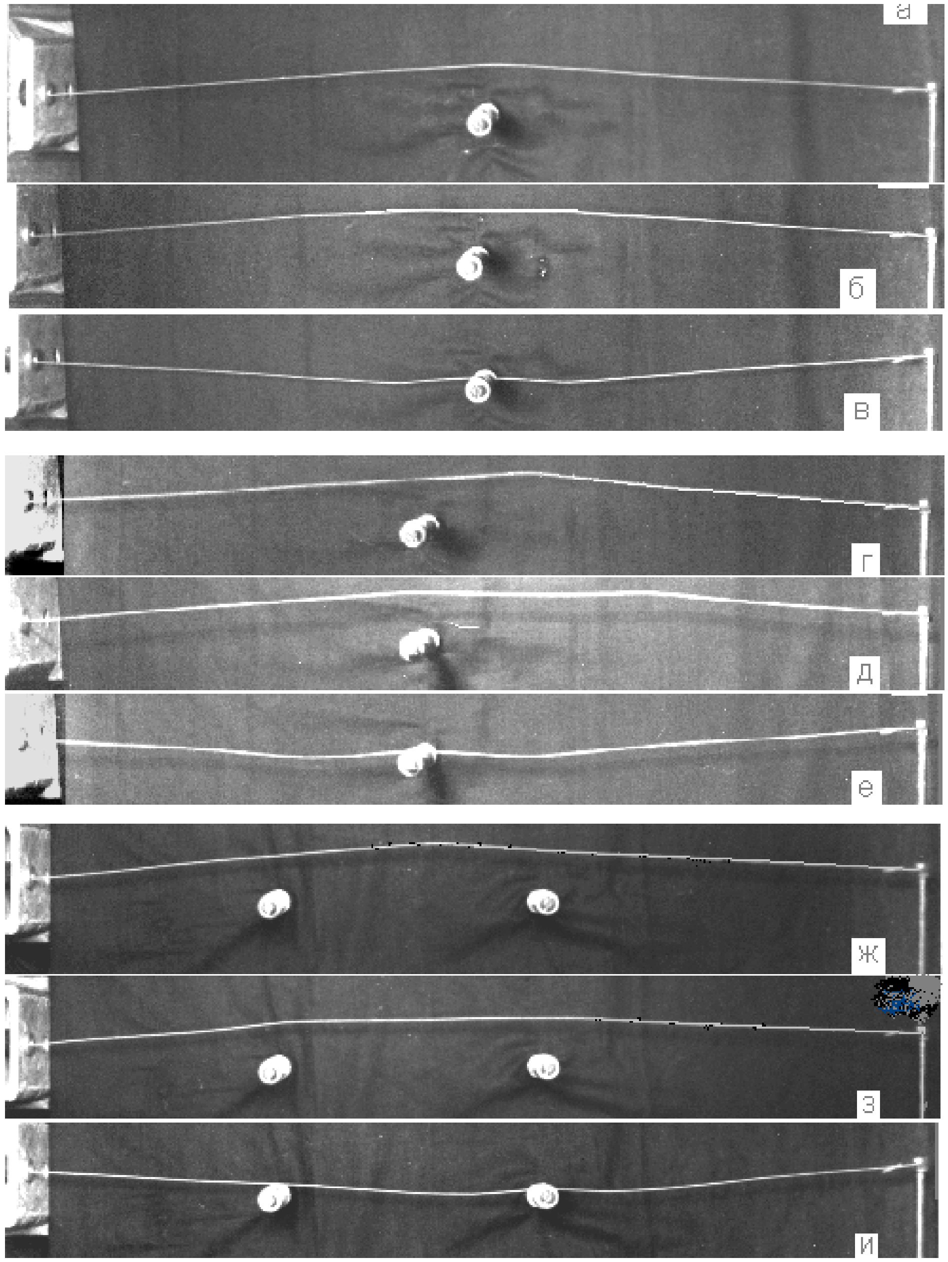

Fig.10

5.2. The experimental study of systems with parallel impact pairs

We can arrive to the concept of distributed impact element by continualizing of discrete systems. An example of impact element obtained in such manner is so called "concentrated beads",

Fig.11

consisting of a string with closely located beads on it. The experiments with such system were carried out at the stand similar to the above described, where a distributed impact element was simulated by a rubber tourniquet with 50 beads of 5 mm diameter located uniformly on it. In this system when passing through frequency range the qualitative character of waves configuration retains: on-running without rebounds in pre-resonance area and puffs in over-resonance area of the linear system. This system's behavior differs in that standing waves were detected in the neighborhood of the linear resonance frequency, given the title "on-running with rebound". At such modes central beads bumping against the restriction rebound immediately, and then the same happens with neighboring beads as they approach the restriction. As this happens the wave forms a configuration, central part of which is expanding progressively along the restriction and has an "inside-out" form (fig.9, b). Such waves arise if excitation intensity extends a certain ultimate level.

A system, containing small number of beads cannot be reduced to a distributed impact element and represents a typical example of an object with parallel impact pairs. Experiments with a "rarefied" system, containing 3 beads, were performed at the same stand, supplied with impact detectors, which present restrictions. In the frequency range $\omega_1<\omega<\Omega_1$ there were "sluggish" modes on record with collisions of the central bead, sometimes accompanied with rattling. In this area there were also irregular vibro-impact modes observed.

In the band $\Omega_1<\omega<\omega_*$ there were recorded stable periodic modes of the claps type with concurrent collisions of all the three beads, which were clearly corroborated by the oscillograms of the signals from the impact detectors.

At further increasing of frequency after passing the third inherent frequency stable confined oscillations of the soliton (breather) type were observed in a thin frequency range.At this mode, which can be obtained by delaying of one-bead oscillations by amplitude, this bead performs intensive oscillations with collisions, while the other two are almost at rest.

In conclusion we present some results of the large set of experiments, conducted at another stand, created for more complete study and systematizing of dynamic conditions which take place in systems with several parallel impact pairs. The stand contains a distributed element with mass-point inclusions - tightened tape made of beryl bronze with ten hardened steel balls rigidly fixed on it at equal spaces. 20 electromagnet exciters integrated with rigid restrictions of the balls movements excite oscillations. The stand enables to change the band length and the number of balls involved, adjust the band tension and clearances. The exciters operate off a sinusoid oscillations generator. The control system enables to re-phase the electromagnets power supply by voltage half-waves or switch specified exciters. The oscillations are monitored by means of the equipment set, containing, in particular, stroboscope movement analyzer, contactless movement detectors and spectrum analyzer.

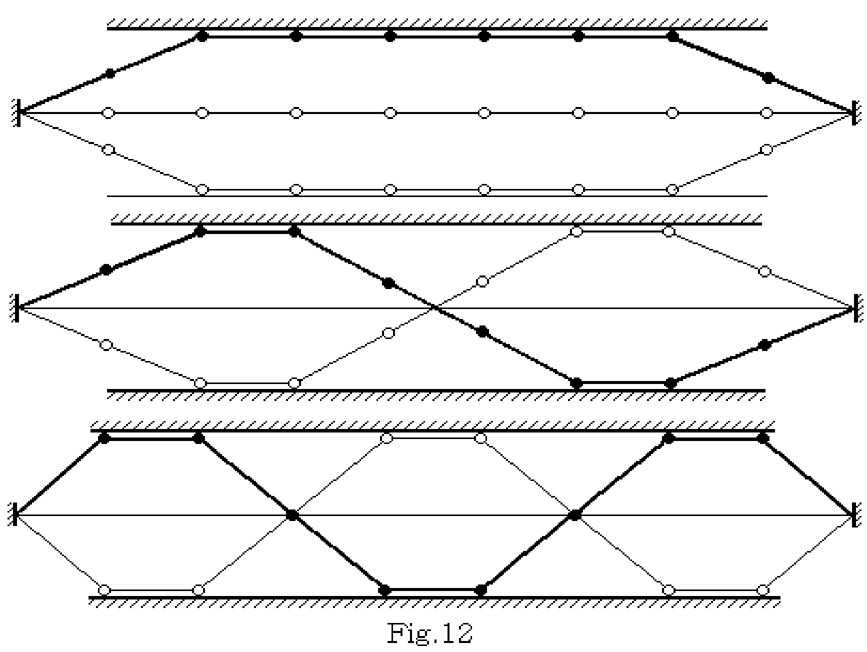

Fig.12

The experiments haven't revealed any types of periodic vibro-impact processes but various forms of puffs and "evident" modes with collisions in a crest of one of the forms, relevant to linear oscillations. And it is the claps which become predominating periodic modes as the number of impact elements grows. Presence or absence of puffs is almost independent of location of the inducing forces points. The experiments have displayed that the effects under consideration are faintly sensitive to a specific kind of exciting action: if an effect was observed at some kind of excitation (adjustment is to be applied to the voltage at the electromagnet winding and its phasing at each exciter), then it was also observed at other excitation types.

Like in systems described earlier, in these experiments trapezoid forms of standing waves were also observed, including "multitrapezoid" forms, composed of trapezoids of one or another polarity. Fig.12 displays the first three trapezoid forms, obtained while observing a system with the number of balls n=8 in stroboscopic lighting. Note, that at realization of such forms, generally speaking, not all the balls are involved in claps.

Profiles of feasible standing waves "constructed" from different trapezoids may be classified by their attribute of "equilaterally". The experiments have shown that feasibility of one or another form of claps depends on the fulfillment of some ultimate ratio of the large base of the trapezoid to the small base, and also on their "equilibrium" - the proportion between projections of the sides on the axis of static equilibrium of the tape. This, particularly, causes the limitation of the minimal number of elements, involved in a puff in a given trapezoid configuration. In studies of symmetric systems with large numbers of impact pairs besides the periodic modes, considered above, there were detected asynchronous movements, arising at a periodic excitation. These modes are usually excited in the neighborhood of the puff forms, for which the above mention feasibility conditions do not fulfill and the force excitation level is enough to take the balls to the restrictions.

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MAIN REFERENCES

General observations. *This text is based on representative enough number of works. To result them in this brief review it is impossible. Detailed lists of references see, in particular, in reviews [1,2]. The general data on used methods can be found in the monograph [3] (see also [4]).*

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